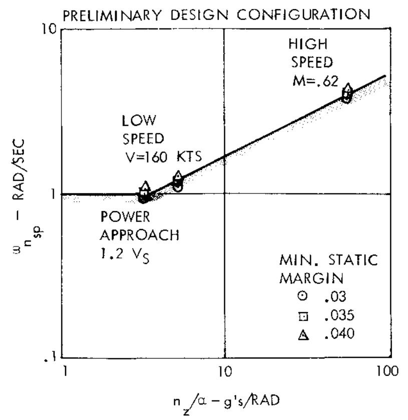

Fig. 3 Short period frequency vs static margin.

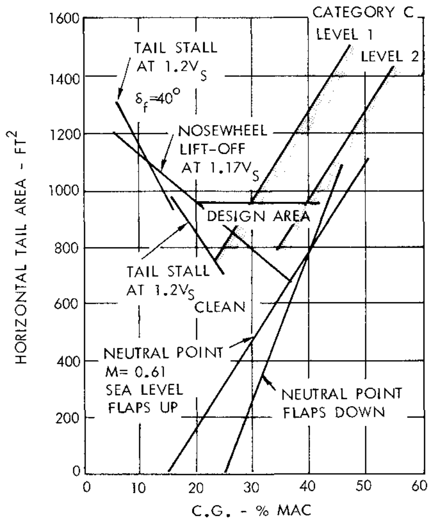

Fig. 4 Effect of minimum short period response on C-5A horizontal tail requirement.

where

$$K_1 = \frac{2l_y}{1.1\, g\rho \mathcal{L}_T^2 (C_{L_\alpha})_H} \qquad K_2 = \frac{W\bar{c}}{I_y}$$

Reference 1 gives different values of the minimum $\omega n_{sp}^2/n_\alpha$ depending on the flight category; however, the relevant one is the largest one, i.e. 0.28 for Level 1, Category A. The smaller required values of CAP are then exceeded automatically. It is also recognized that just meeting this minimum requirement does not guarantee acceptable flying qualities. It is believed to be adequate for preliminary design use, however.

Figure 2 shows the use of the suggested procedure on a fighter configuration needing a 20% C.G. range compared with using an arbitrary 3% minimum static margin. For this design there is only a small change in required tail volume coefficient, but the minimum design static margin is indicated immediately as 3.5% means aerodynamic chord. Figure 3 represents the results of a full three-degree-of-freedom calculation of longitudinal dynamic stability for the same configuration. Static margin was varied for power approach, low speed, and high speed conditions. The results show that in order to meet the minimum frequency criterion a minimum static margin of 3.4 to 3.9% is required throughout the speed range. Although another criterion is obviously required for the region where $\omega_{n_{sp}} > 1$, these results are in excellent agreement with the simple procedure proposed in this Note.

By contrast, Fig. 4 shows the results of the procedure applied to the C-5A configuration. Category A of Ref. 1 is assumed not to apply to this type of aircraft, the appropriate criteria are those of Category C ($\omega_{n_{sp}}^2/n_\alpha$ of 0.16 for Level1 and 0.096 for Level 2). It would obviously be impractical to meet the Level 1 requirements without a stability augmentation. Thus, in the earliest design phase, the procedure indicates a requirement for, and probable level of, a stability augmentation system.

III. Conclusions

A procedure is suggested for determining the aft center of gravity limit required to meet the flying qualities specification for short period dynamics. It is recognized that the procedure does not necessarily guarantee acceptable flying qualities. It does form a rational method for calculating the aft center of gravity limit in the initial design phase of sizing the horizontal tail of a new airplane configuration. For configurations where this criterion is unduly restrictive, the developed procedure immediately indicates the need for a stability augmentation system.

References

[1] Anon., "Military Specifications. Flying Qualites of Piloted Airplanes," MIL-F-8785B (ASG), Aug. 1969.

Technical Comments

Comment on "Solution of the Lifting Line Equation for Twisted Elliptic Wings"

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FILOTAS[1] has given an explicit solution for the case of a sinusoidally twisted elliptic wing using the Prandtl lifting line equation in which the circulation distribution is expanded in an infinite series of Chebyshev polynomials of the second kind. The intention of this Note is to show that a simpler explicit solution exists for the conventional sine series expansion for the circulation, Γ

$$\Gamma(\theta) = \sum_{n=1}^{\infty} A_n \Gamma_n(\theta) = 2sU \sum_{n=1}^{\infty} A_n \sin n\theta \tag{1}$$

where s is the wing semispan, U the freestream velocity, A_n are constants, $\theta = \cos^{-1}(y/s)$ measured from wing tip to tip, and y is the spanwise coordinate positive on the right wing.

In Ref. 2, an integral form of the solution is derived for the general case of a twisted wing which reads [Ref. 2, Eq. (12)]

$$\frac{\pi}{2} sUnA_n + (1/\pi U) \int_0^\pi [\Gamma_n(\theta) \sum_{r=1}^{\infty} A_r \Gamma_r(\theta)/c(\theta)] \sin\theta \, d\theta = \int_0^\pi \alpha(\theta) \Gamma_n(\theta) \sin\theta \, d\theta \tag{2}$$

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where $\alpha(\theta)$ is the spanwise angle-of-attack distribution. For an elliptic wing the chord distribution is $c(\theta)=c_o\sin\theta$ where c_o is the length of the ellipse minor axis, and the unknown constants A_n can then be solved for explicitly for any twist distribution. The sinusoidal twist

$$\alpha(\theta)=\sum_{r=1}^{\infty}\alpha_r\sin r\theta+\sum_{r=1}^{\infty}\beta_r\cos r\theta \tag{3}$$

in particular, gives the solution

$$\left(n+\frac{4s}{\pi c_o}\right)A_n=-\frac{16n}{\pi}\sum_{r=1}^{\infty}{}' r\alpha_r/[(r+n)^2-1][(r-n)^2-1]+\sum_{r=1}^{\infty}\beta_r d_r \tag{4}$$

where

$$\begin{aligned} d_r &= 1;\ \text{if}\,(r-n)=1\\ &=-1;\ \text{if}\,(n-r)=1\\ &=0 \quad \text{otherwise}\end{aligned} \tag{5}$$

and Σ' denotes summation only over even $(n+r)$. This result is simpler than those of Ref. 1 which contains Bessel functions of the first kind and Chebyshev polynomials of the second kind. The flat plate result is well known and may be added to the solution.

References

[1] Filotas, L. T., "Solution of the Lifting Line Equation for Twisted Elliptic Wings," *Journal of Aircraft*, Vol. 8, Oct. 1971, pp. 835-836.

[2] Bera, R. K., "Some Remarks on the Solution of the Lifting Line Equation," *Journal of Aircraft*, Vol. 11, Oct. 1974, pp. 647-648.

Comment on Papers by B. W. Roberts and K. R. Reddy

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PUBLISHED simultaneously by members of the same university department, these two papers are obviously closely related, and it seems permissible to submit one comment on those aspects which are generally common to both.

Roberts' specific criticisms of filling time concepts† are certainly well taken. The fact that they were put forward in the first place is a quirk of history, in that the first parachutes were developed empirically, and to a certain extent, a philosophy of "cut and try" has pervaded the field ever since. There has been little motivation to put scarce R&D money into theoretical studies because: a) existing theoretical predictions were hopelessly far removed from experimental observation[6]; b) "cut and try" is not prohibitively expensive; and c) although not complex by comparison with many aerodynamic problems which we have solved satisfactorily in the past, the opening process cannot be solved by a single "Ph.D. thesis" level of effort. A properly planned, multifaceted attack on the problem is needed, for which, as we have suggested, there is little official motivation.

The papers of Roberts[1] and Reddy[2] are to be welcomed as serious preliminaries on one aspect of the inflation process, but the reader might be tempted to read more into these papers than the authors perhaps intended. Apart from the fact that the three-dimensional problem is treated as two-dimensional, a number of important terms are omitted from the analysis. Most of these were at least outlined in Ref. 7,‡ a paper which has apparently escaped the attention of both authors. The most important considerations omitted would seem to be as follows:

1) When the canopy is opening, it moves toward the payload. In the simplest case, treated by Reddy, the distance between the store and the wedge apex is, in Reddy's notation

$$X=b\cos\alpha+L\cos\delta=b\cos\alpha+[1-(b/L)^2\sin^2\alpha]^{1/2} \tag{1}$$

If V_s is the store's velocity along the flight path, and if the store mass greatly exceeds the virtual canopy mass the velocity of the canopy is

$$V_c=V_s+b\sin\alpha\frac{d\alpha}{dt}\left[1+\frac{1}{[1-(b/L)^2\sin^2\alpha]^{1/2}}\right] \tag{2}$$

Note that as $\alpha\to 0$ and/or $b/L\to 0$

$$V_c\to V_s+b\sin\alpha(d\alpha/dt) \tag{2a}$$

With the same assumptions, the canopy acceleration is

$$\frac{dV_c}{dt}=\frac{dV_s}{dt}+b\left[1+\frac{1}{[1-(b/L)^2\sin^2\alpha]^{1/2}}\right]\left[\left(\frac{d\alpha}{dt}\right)^2\cos\alpha+\frac{d^2\alpha}{dt^2}\sin\alpha\right]+\frac{b\sin^2\alpha(b/L)^2(d\alpha/dt)^2}{[1-(b/L)^2\sin^2\alpha]^{3/2}} \tag{3}$$

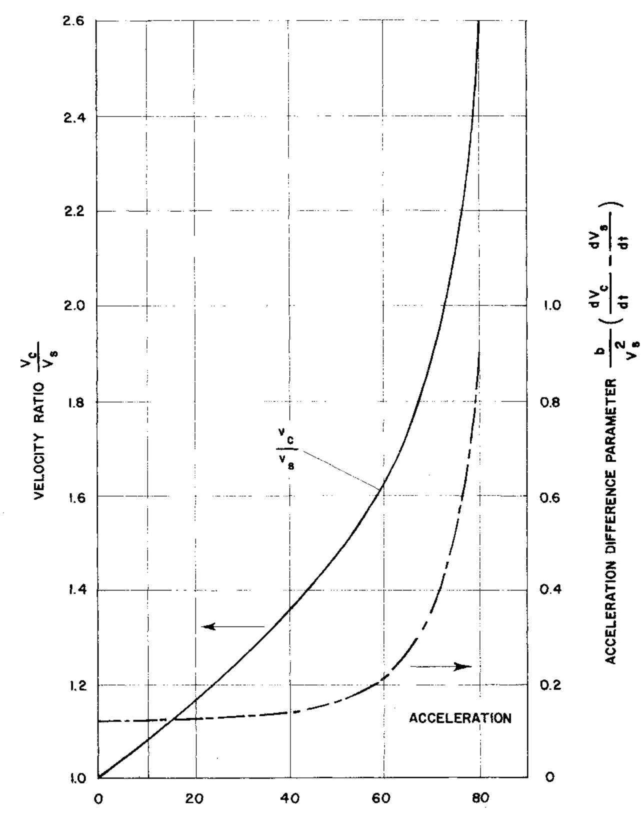

Fig. 1 Canopy velocity and acceleration parameters for infinite payload mass, inextensible shrouds, constant angular inflation velocity and $\alpha_f=70°$ $(U_0t_f/b)=5.0$, $b/L=1.0$.

Index categories: Aircraft Deceleration Systems; Nonsteady Aerodynamics.

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†References 3-5 are examples he cited; more are cited in Ref. 7.

‡Reference 7 was submitted in November 1970, published February 1973 with the legend "Received June 1972."